

*Relationships among Central Angles, Arcs, and Chords*  
Day 1 Student Task

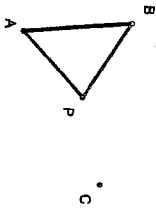
In this unit you will study properties of circles. As you progress through the unit, you will learn new definitions and develop and prove numerous theorems. It will be advantageous for you to keep a "Circle Book" that includes the definitions and theorems addressed in each task. With each definition and theorem you enter, you should also include an illustrative sketch.

We will begin by re-visiting the definition of a circle.

1. Use a compass to construct a circle on an unlined sheet of paper. Label the center of your circle.
  - a. What information do you need to determine a unique circle?
  - b. Use your answer to *Item a* to help you write a definition of a circle.

Now we will introduce some notation and terminology needed to study circles. Consider the figure at right.

$$m\angle APB = 75^\circ$$



Circles are identified by the notation  $\odot P$ , where  $P$  represents the point that is the center of the circle.

A **central angle** of a circle is an angle whose vertex is at the center of the circle.  $\angle APB$  is a central angle of  $\odot P$ .

A portion of a circle's circumference is called an **arc**. An arc is defined by two endpoints and the points on the circle between those two endpoints. If a circle is divided into two unequal arcs, the shorter arc is called the **minor arc** and the longer arc is called the **major arc**. If a circle is divided into two equal arcs, each arc is called a **semicircle**.

In our figure, we call the portion of the circle between and including points  $A$  and  $B$ , arc  $AB$  notated by  $\widehat{AB}$ . We call the remaining portion of the circle arc  $ACB$ , or  $\widehat{ACB}$ . Note that major arcs are usually named using three letters.

We say that the central angle  $\angle APB$  *intercepts* or has  $\widehat{AB}$ . We also say that  $\widehat{AB}$  *subtends* or has the central angle  $\angle APB$ . Note that when we refer to the arc of a central angle, we usually mean the minor arc unless otherwise stated.

Arcs are measured in two different ways - using degree measure and using linear measure. Usually when we refer to the measure of an arc, we are referring to the degree measure. The measure of a minor arc is defined to be the measure of the central angle that intercepts the arc. The measure of a major arc is  $360^\circ$  minus the measure of the central angle that intercepts the arc. In the figure above, the measure of  $\widehat{AB}$  is  $75^\circ$  because that is the measure of its central angle. The measure of  $\widehat{ACB}$  is  $360^\circ - 75^\circ$  or  $285^\circ$ .

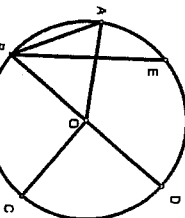
The length of an arc is different from its measure. The length is given in linear units and is determined as a portion of the length of the entire circumference of the circle. We will investigate the length of an arc in a later task. **Congruent arcs** have equal degree measures and equal lengths.

A **chord** is a *segment* whose endpoints lie on the circle. In the above figure, segment  $\overline{AB}$  is a chord of  $\odot P$ .

2. How many chords can be in a circle?

What is the longest chord in a circle? Explain how you know?

3. Refer to the figure at the right. Identify and name each of the following. Be sure to use the correct notation.
  - a. Two different central angles
  - b. A minor arc
  - c. A major arc
  - d. A semicircle
  - e. Two different chords
  - f. The central angle subtended by  $\widehat{AD}$



Use your protractor to help you find the following measures:

- g. The measure of  $\widehat{AC}$
- h. The measure of  $\widehat{DEC}$